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THE UNIVERSITY OF ALBERTA

SOME EXPERIMENTS ON  
SUPERFLUIDITY IN LIQUID HELIUM II

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES IN  
PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE  
OF MASTER OF SCIENCE

DEPARTMENT OF PHYSICS

by

Asif Uddin Hassan

EDMONTON, ALBERTA

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ACKNOWLEDGEMENTS

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I should like to express my thanks to the National Research Council for the financial assistance received for undertaking this project.



## ABSTRACT.

The experiments described in this thesis were carried out in order to design an experimental arrangement using second sound in liquid helium II to excite a resonator analogous to a Helmholtz resonator in a gas. Part I describes the experiments on oscillations in the flow of liquid helium II at temperatures  $1.3^{\circ}\text{K} - 2.01^{\circ}\text{K}$ . Only isothermal oscillations in the case of helium II film type superleak were carried out successfully. These experiments were undertaken to design a suitable resonator. Due to the failure of getting a wire-filled-tube type superleak properly made further experiments on oscillations could not be carried out. Part II describes the experimental arrangements designed to generate the second sound in a reversible manner.



## I N T R O D U C T I O N .

The object of the work described in this thesis is to develop the necessary experimental techniques for an experiment using second sound in liquid helium II to excite a special type of resonator. The resonator in this case is an analogue of a Helmholtz resonator in a gas corresponding to a resonator in the superfluid in helium II. The object of such an experiment (first suggested by Manchester, 1957) is to use second sound to excite such a superfluid resonator and from measurements of the response characteristics of the resonator to obtain information on the flow of helium II through the superleak which constitutes the "neck" of the resonator. A similar experiment for a resonator with a neck rather wider than a superleak has been reported by Kramers (1960). The two principal requirements for this experiment are the provision of a suitable resonator and a second sound source.

The resonator must be adiabatic and have a superleak of well defined geometry (Manchester 1957) and the second sound source must not produce a net input of heat to the helium II bath - in other words, the second sound must be produced reversibly. If this were not so then



the condition of temperature equilibrium for the contents of the resonator and surrounding bath could not be maintained and therefore the experiment could not be carried out under the stable conditions necessary.

The first part of this thesis will be concerned with a discussion of the oscillations in the flow of liquid helium II, the second part will describe the generation of second sound reversibly. An experiment to "drive" the resonator has not yet been carried out.





PART I.

OSCILLATIONS IN LIQUID HELIUM II.



## CHAPTER I

### OSCILLATIONS IN LIQUID HELIUM II.

At temperatures below the  $\lambda$ -point, liquid helium (known as Helium II) exhibits some remarkable properties, the most important of which is the "superfluidity" discovered by Kapitza (1938) ; Allen and Misener (1938). According to the theories of Tisza (1938, 1947) and Landau (1941) the properties of helium II can be described from a phenomenological standpoint by considering it as a mixture of two types of fluid. - "normal" fluid, with ordinary liquid properties and "superfluid" which is assumed to carry little or no entropy, and to be almost entirely non-viscous.

Allen and Misener (1938), in the course of their pioneering experiments on the properties of the superfluidity of liquid helium II, first discovered the oscillations of a liquid helium level in a container connected to the helium bath via a superleak (a narrow channel through which only the superfluid could flow). Atkins (1950) reported a similar type of oscillation during his experiments on the measurements of the thickness of the helium II film. As the temperature difference between the container and the bath in these experiments was very small (of the order of  $10^{-6}$  K) these are known as "isothermal oscillations".



Robinson (1951) predicted an adiabatic type of oscillation if the container and bath could be thermally isolated. Manchester (1955) successfully carried out experiments to show the existence of these adiabatic oscillations.

A brier outline of the theory of oscillations in liquid helium II will be given in this chapter.

### "ISOTHERMAL OSCILLATIONS"

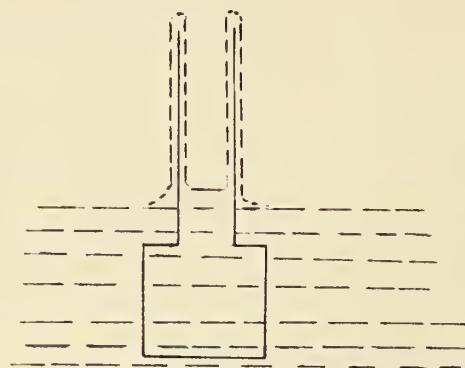
In a system consisting of a vessel containing liquid helium II and connected to a liquid helium bath via a superleak, isothermal oscillations occur if good thermal contact is maintained between the two. Though it was in the early experiments of Allen and Misener (1938) that these oscillations were first observed it was not until Atkin's (1950) experiments that care was taken to ensure good isothermal conditions.

In order to understand the mechanism of these oscillations, consider a vessel containing liquid helium II immersed in a liquid helium bath and emptying through the film (Fig.A). The two levels of liquid helium inside the vessel and outside in the bath eventually achieve the same height, but the rate of transfer is almost independent of the level difference, so the film is still moving rapidly when near the bath level and its momentum causes the level to overshoot and oscillate about



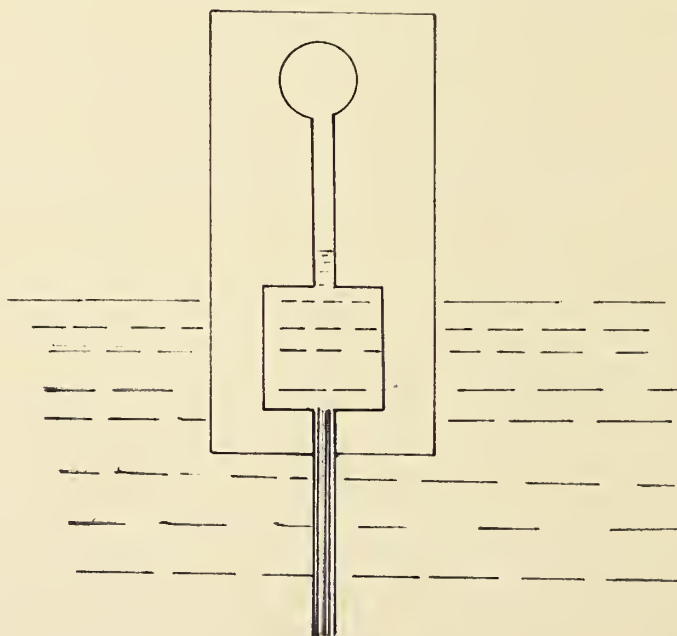


FIGURE.(A)



ISOTHERMAL OSCILLATIONS FOR FILM.

FIGURE.(B)



VESSEL FOR ADIABATIC OSCILLATIONS



its equilibrium position. This oscillation can be regarded as a periodic interchange of energy between the kinetic energy of the film and the potential energy of the liquid inside and outside the vessel. Atkins made the assumptions that the velocity in the film at a fixed height is independent of the distance from the wall, and that the thickness of the film is independent of its velocity. With these assumptions the frequency of an oscillation was found to be

$$\omega = \left[ \frac{\rho_s \sigma g}{\rho r \left(1 + \frac{r}{R}\right) \int_0^l \frac{dH}{\alpha}} \right]^{1/2}$$

where "r" and "R" are the inner and outer radii of the beaker and "l" is the height of the rim of the beaker above the bath level,  $\rho_s$  and  $\rho$  are the densities of the superfluid and bulk fluid, "g" is the acceleration due to gravity, "d" is the film thickness at a given height "H" above the surface of the bath.

This value of the frequency of an oscillation is consistent with the frequency of the isothermal oscillations in case of the flow through a superleak of fixed geometry such as a wire-filled-tube (as described in chapter II ) as investigated by Robinson (1951)

$$\omega_i = \left[ \frac{\rho_s \sigma g}{\rho l a} \right]^{1/2}$$



where " $\sigma$ " is the cross-sectional area of the superleak " $l$ " is its length and " $a$ " the cross-sectional area of the observational capillary and the rest of the symbols have the same meaning as in the Atkins' relation.

### "ADIABATIC OSCILLATIONS"

In his analysis of oscillations in liquid helium II, Robinson (1951) suggested that if the vessel and bath were isolated, then it would be mainly the "fountain effect" (Allen and Jones 1938) and not gravitation which determines the frequency of the oscillations.

The adiabatic oscillations can be considered with the help of the Fig. B. The thermal contact between the inside vessel and the bath of liquid helium is reduced to a minimum by surrounding the experimental vessel with a vacuum jacket. A superleak similar to Allen and Misener's (1939) joins the inside vessel with the helium bath. Temperature differences between the inside vessel and bath liquid arise due to the flow of liquid from one to the other. If the liquid (say) flows from inside the vessel to the bath, the temperature of the inside vessel rises because of the mechano-caloric effect. This sets the temperature gradient in such a direction that



the thermo-mechanical effect tends to force the liquid back into the vessel. As the fluid moving through the superleak has sufficient momentum, oscillations set in.

The frequency of these oscillations given by Robinson (see the original paper for details, 1951) was

$$\omega_a = \omega_i (1 + \alpha)^{\frac{1}{2}}$$

$$\text{where } \alpha = \frac{a T S^2}{g V C} \left[ 1 - \frac{1}{\rho S} \left( \frac{dP}{dT} \right)_{\text{vap}} \right]$$

"S" and "C" are the entropy and specific heat respectively of liquid helium II, "V" is the volume of the liquid helium inside the container and the other symbols have the same meaning as before.  $(dp/dT)_{\text{vap}}$  is the gradient of the vapour pressure curve, and T is the absolute temperature. In his analysis Robinson divided the solution of the differential equation for the frequency into three different regions, depending on how good was the thermal contact between the vessel and the bath. These three regions are shown in Fig. 4 (d), viz the "isothermal", "aperiodic" and "adiabatic" regions.

The relation for the frequency of adiabatic oscillations may also be written as

$$\omega_a = u_2 ( \rho_n \sigma / \rho \lambda v )^{\frac{1}{2}}$$

if the effects of gravitation and the term involving the



gradient of the vapour-pressure curve are ignored. The second sound velocity is given by

$$u_2^2 = \rho_s T s^2 / \rho_n c$$

This expression for  $\omega_a$  is analogous to the expression for the resonant frequency of a Helmholtz resonator in first sound and can be thought of as giving the resonant frequency of a second sound resonator. Thus it is possible in principle to excite this system of insulated container and superleak to resonance by "driving" it with an external second sound source and thus to observe the nature of its amplitude vs frequency characteristics. If this could be done then the flow of helium through the superleak could be examined in a rather unusual way. Quantative results on the damping of the system could be used to examine the applicability of several types of equations of motion which have been suggested by different theoretical approaches to the problem (see Daunt and Smith, 1954). The resonator analogue involves adiabatic oscillations only but some measurements on isothermal oscillations were made in the hope that they would supply data on the flow characteristics of the superleak used. These measurements did not give a straight forward answer as is discussed in Chapter IV.





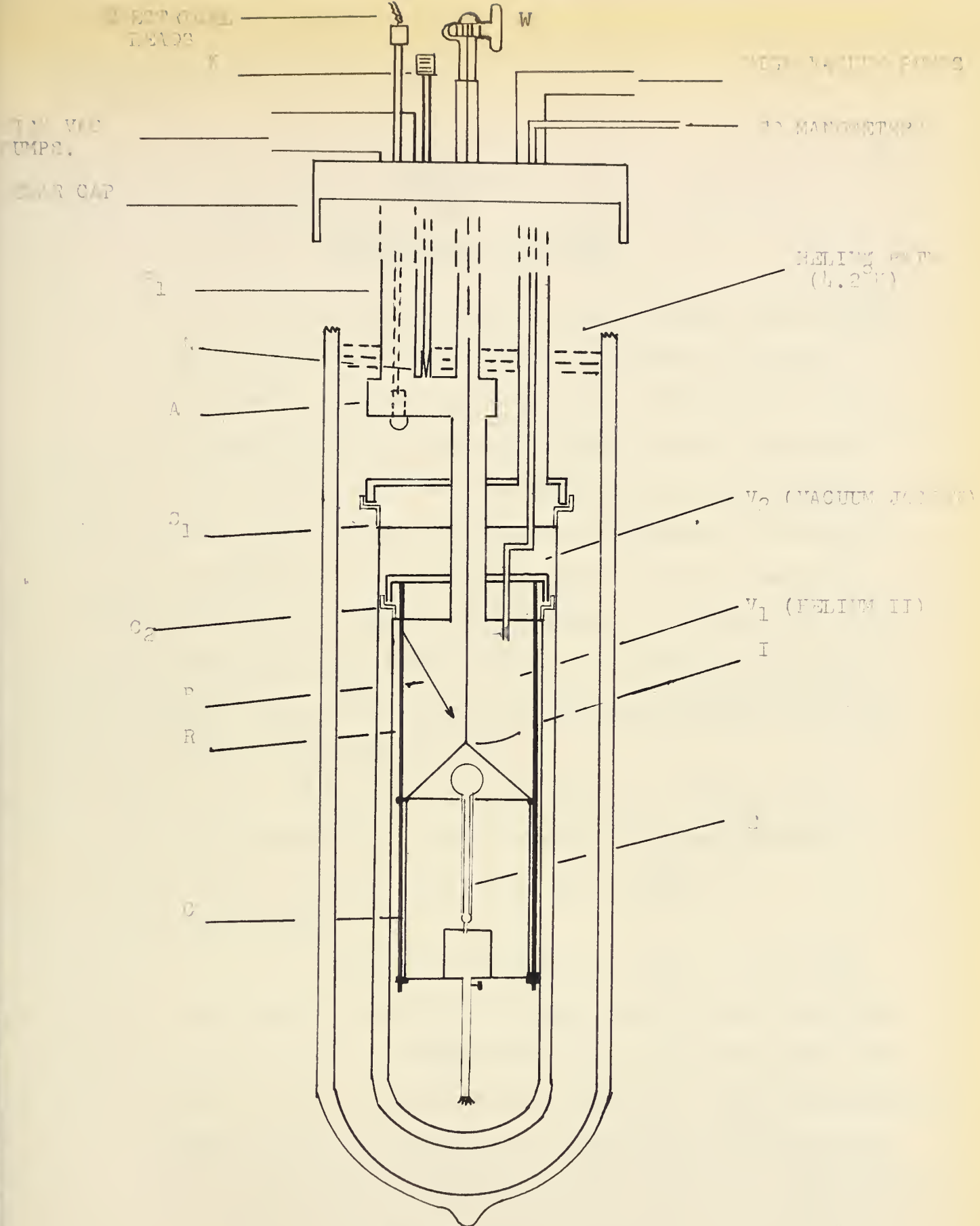


FIGURE 1

SCHEMATIC OF THE "COLD" PART OF THE APPARATUS



## CHAPTER II

### EXPERIMENTAL APPARATUS

The experiments designed to study the isothermal or adiabatic oscillations discussed in Chapter I required a cryostat in which visual observations could be made and with little alteration the same cryostat could be used for the reversible generation of second sound. This would then allow the experiment involving a resonator driven by second sound to be performed. Two experimental vessels were designed, one making use of an artificial superleak - a wire filled tube (Manchester 1955), and the other that of natural superleak viz the helium film (Atkins 1950).

A detailed account of the characteristics of the cryostat and the experimental vessels designed is give in this chapter.

#### THE CRYOSTAT.

The cryostat designed (Fig. 1 ) had the same general features as in any other cryostat used for work between the temperatures  $1^{\circ}\text{K}$  -  $4.2^{\circ}\text{K}$ . The unique feature of this cryostat was the use of two large copper-glass seals  $C_1$  and  $C_2$ . The copper to glass seals enabled glass to be used for the walls of the two chambers - the vacuum



jacket and the experimental chamber. The inner seal  $C_2$   $2\frac{1}{8}$ " O.D. and  $14\frac{1}{2}$ " long was used as the experimental chamber. The outer seal  $C_1$   $2\frac{3}{4}$ " O.D. and  $16\frac{1}{2}$ " long was used as the vacuum jacket  $V_2$  to isolate the experimental vessel E from the helium bath at  $4.2^\circ\text{K}$ . This helium bath was a liquid helium dewar with 80mm I.D. and overall length of 88 cm. This was shielded by liquid air in a dewar of 125 mm I.D. and overall length of 70 cm.

The experimental vessel (a detailed account to be given later) was mounted on the carriage C and was moved up and down by means of the winch W and fishing line I, being guided by the pillars R. A needle P was soldered on one of the pillars so that when after finishing an experiment the carriage was pulled up all the way, the bulb B (Fig. 2) could be broken.

The pumping lines were made of  $3/4$ " and  $3/8$ " thin walled inconel tubing for  $V_1$  and  $V_2$  respectively. These were used to minimize heat leakage due to the thermal conduction from the room temperature end of the tubes. To avoid radiation down the pumping tube to the helium chamber  $V_1$  the pumping tube was broken by inserting a small rectangular brass box A. Below this radiation trap the pumping line was  $\frac{1}{2}$  inch inconel tubing. All joints between



the metal pieces were connected by brass ferrules and were soft soldered. The glass jackets were soldered in place with a Wood's metal joint between the copper part and the metal body of the cryostat. (See the photograph on page 14).

The helium was transferred by opening the needle valve N operated with the help of knob K. By reducing the vapour pressure over the boiling liquid helium the temperature could be varied between  $4.2^{\circ}\text{K}$  -  $1.3^{\circ}\text{K}$ . This was done by pumping through the line ( $P_1$ ).

For measuring the vapour pressure inside the chamber  $V_1$  a mercury and an oil manometer were connected by a stainless steel tube of 1/16 inch O.D. passing through the 3/8 inch O.D. inconel tube used for pumping over the jacket  $V_2$ . This vacuum jacket around the tube was provided to minimize the thermal disturbance from outside which could affect the vapour pressure reading. (See Clement, Logan and Gaffney Phys. Rev. 100, 743 (1955)).

A mechanical pump with a speed of 100 litres per minute was used for securing the low vapour pressure in the helium chamber. The vacuum jacket  $V_2$  was evacuated with a "Speedivac" oil diffusion pump of speed 80 litres per second.







"GLASS JACKETS SHOWING EXPERIMENTAL CHAMBER"



In order to measure the high vacua in  $V_1$  and  $V_2$  a Phillips type cold cathode ionization gauge (H.Martin and Co.Chicago) was used. Typical values achieved over a number of runs were - at room temperature  $1 - 3 \times 10^{-6}$  mm of Hg. and at liquid air temperature  $1 \times 10^{-6}$  mm to  $8 \times 10^{-7}$  mm of Hg. and at liquid helium temperature  $8 - 4 \times 10^{-7}$  mm of Hg.

#### EXPERIMENTAL VESSELS.

To observe the isothermal type of oscillations two different vessels (Figs. 2 and 3) were made. We call them (a) the Film Type and (b) Superleak Type.

##### (a) FILM TYPE VESSEL.

An arrangement similar to that described by Atkins (1950) was used. In the present case the dimensions of the vessel C (Fig.2) were 2.1 cm long, 2 cm in diameter, base thickness 0.023 inch, top 0.016 inch thick and the wall was 0.006 inch thick. This was joined by means of a  $1/8$ " diameter kovar seal K to a glass capillary of 0.61 mm I.D. The walls of the capillary were flared out to smoothly match to those of a tube of larger diameter (I.D. 2.4 mm) which dipped into helium bath.

##### (b) "SUPERLEAK" TYPE VESSEL.

A copper cannister of the same dimensions as given in (a) was made out of a copper piece



FIGURE (2)

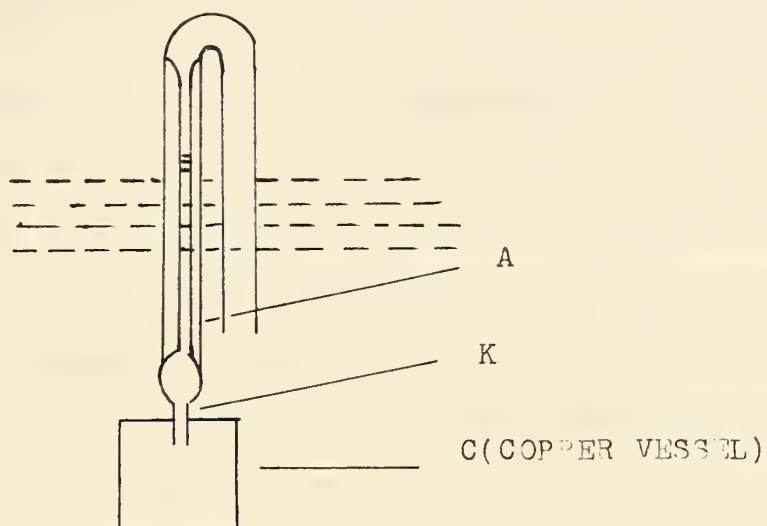
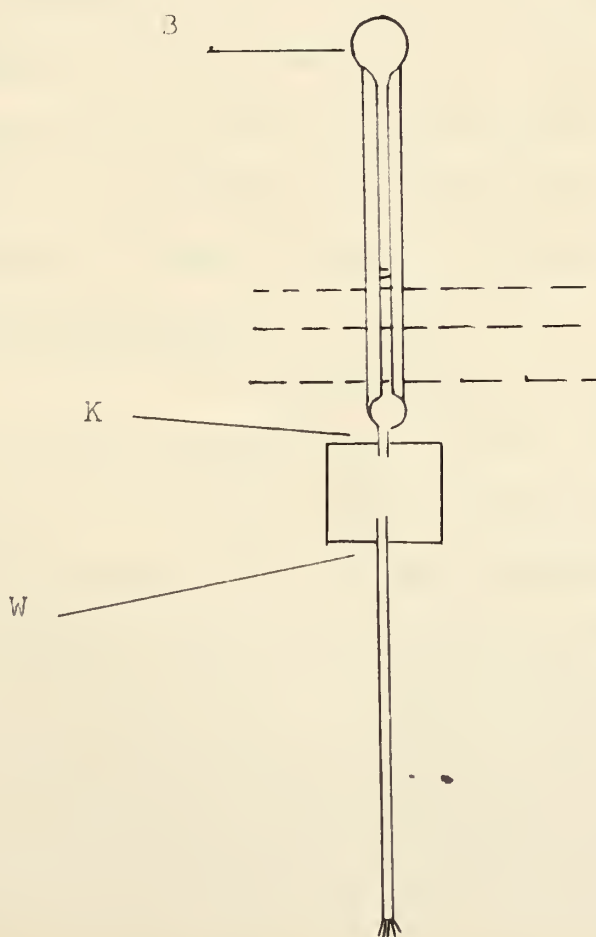


FIGURE.( 3)



WIRE-FILLED-TUBE VESSEL. ( SUPERLEAK )



and the base thickness was reduced to 0.016 inch. A wire-filled-tube (details to be given in Appendix I) made at U.B.C by Manchester (1955) was used as a superleak and was soft soldered at W (Fig.3). A capillary A (I.D. 0.5 mm) similar to the one used in (a) was slightly modified by having the upper end closed by the bulb B. This precaution was taken to avoid any contribution to the observed flow from the helium II film. The bulb B was such that it could be easily broken while the apparatus was being warmed up after an experiment. This vessel was placed in the carriage C (Fig. 1) in place of the vessel (a). As the temperature inside the experimental chamber dropped below the  $\lambda$ -point the superfluid part of the helium travelled through the channels of the superleak and filled the copper cannister. After some two hours of pumping over the helium bath  $V_1$  (Fig.1) and holding the temperature at  $1.6^\circ\text{K}$  (say) the superfluid became visible in the capillary.

#### ELECTRICAL LEADS.

The electrical leads out of the cryostat's experimental chamber were about a meter long and made from B and S, # 40 gauge, single formvar, cotton insulated, copper wires. To allow for the heat influx through the wires due to thermal conduction, the wires were thermally anchored at





A(Fig.1).This anchor was made from a copper plug which was electrically insulated with G.E. varnish (baking type). The wires were wound a few times on the cylindrical plug and held in position with more varnish.The plug was then put in place and soldered in with Wood's metal.

#### GENERAL BEHAVIOUR OF THE CRYOSTAT.

Once all the metal joints were properly soldered the two vacuum jackets gave good high vacua within a reasonable time of pumping.In the early runs it was noticed that the rate of loss of liquid helium was very high and this was caused by some oscillations in the helium gas column (see Keesom's book on Helium p 97) giving a humming noise.This was remedied by soldering copper discs at two places along the pumping lines of the two jackets.Then as the helium was transferred into the inside system (experimental chamber) the oscillations set in again. These were removed by pumping while transferring.The oscillations disappeared but as soon as the liquid helium reached some certain level the oscillations set in again. These were removed by pumping the space at the top of the tube which allowed the fishing line carrying the carriage to be raised or lowered.This removed the static column of helium gas in the tube and this seemed to be the column of gas which was oscillating.It seems therefore that all



static columns of helium gas in a cryostat should either be eliminated or as in the case of the vapour-pressure tube they should be surrounded with a vacuum jacket.

In the earlier trial experiments a carbon resistance (Allen and Bradley 10 ohm resistor placed inside the experimental chamber at the bottom) was also used to measure the temperature changes, in addition to the vapour pressure measurements. As the liquid level inside the experimental chamber fell so low that the experimental vessels were above the level, it became necessary to transfer more liquid inside the experimental chamber from the liquid helium bath ( $4.2^{\circ}\text{K}$ ) to the already pumped liquid ( $1.3^{\circ}\text{K}$ ). The mixing of these two liquids exhibited a peculiar type of oscillations recorded with the help of the carbon resistance thermometer.

The liquid at the bottom warms up to the  $\lambda$ -point. The upper part of the liquid started to cool down slowly as pumping was started. Oscillations in the temperature occurred just when the temperature of the upper portion reached the temperature of the lower section i.e., approximately at the  $\lambda$ -point. These oscillations died out when the temperature of the entire liquid began to fall down.



### CHAPTER III.

#### EXPERIMENTAL PROCEDURE.

The vacuum space was checked thoroughly to give good thermal insulation before an experiment was carried out. This was done pumping overnight. Rapid cooling could produce undue strain on the copper-glass seals, so extreme care was taken in cooling the system. The outer parts of the apparatus were first cooled by immersing them in a dewar of liquid air for a few hours. Helium exchange gas was introduced into the two vacuum jackets  $V_1$  and  $V_2$  to help the interior parts cool down to liquid air temperature.

After this pre-cooling liquid helium could be transferred. This was done by quickly emptying the liquid air from the dewar which was then replaced on the apparatus, sealed at the top, and surrounded by another dewar containing liquid air. The inner dewar was connected to the automatic helium recovery system. When the liquid level was some couple of inches above the needle valve N (fig.1), the helium was transferred into the experimental chamber, by opening the valve with the knob K. By pumping away the exchange gas from  $V_2$  the thermal contact between the helium bath and the inner chamber was



broken and lower temperatures were achieved by pumping over the helium in the experimental chamber. After some two hours of pumping the temperature usually fell to  $1.3^{\circ}\text{K}$ .

The liquid helium used in all experiments was produced by the Collin's liquefier in the Physics Department at the University of Alberta.

#### VAPOUR PRESSURE MEASUREMENTS.

Two manometers mounted on a board at one side of the cryostat were used to measure the vapour pressure in the helium experimental chamber. These were continuously evacuated on the reference side. The T 55 E temperature scale defined by Clement, Logan and Gaffney (1955) was used to convert the vapour pressure readings to absolute temperatures.

#### OSCILLATIONS MEASUREMENTS.

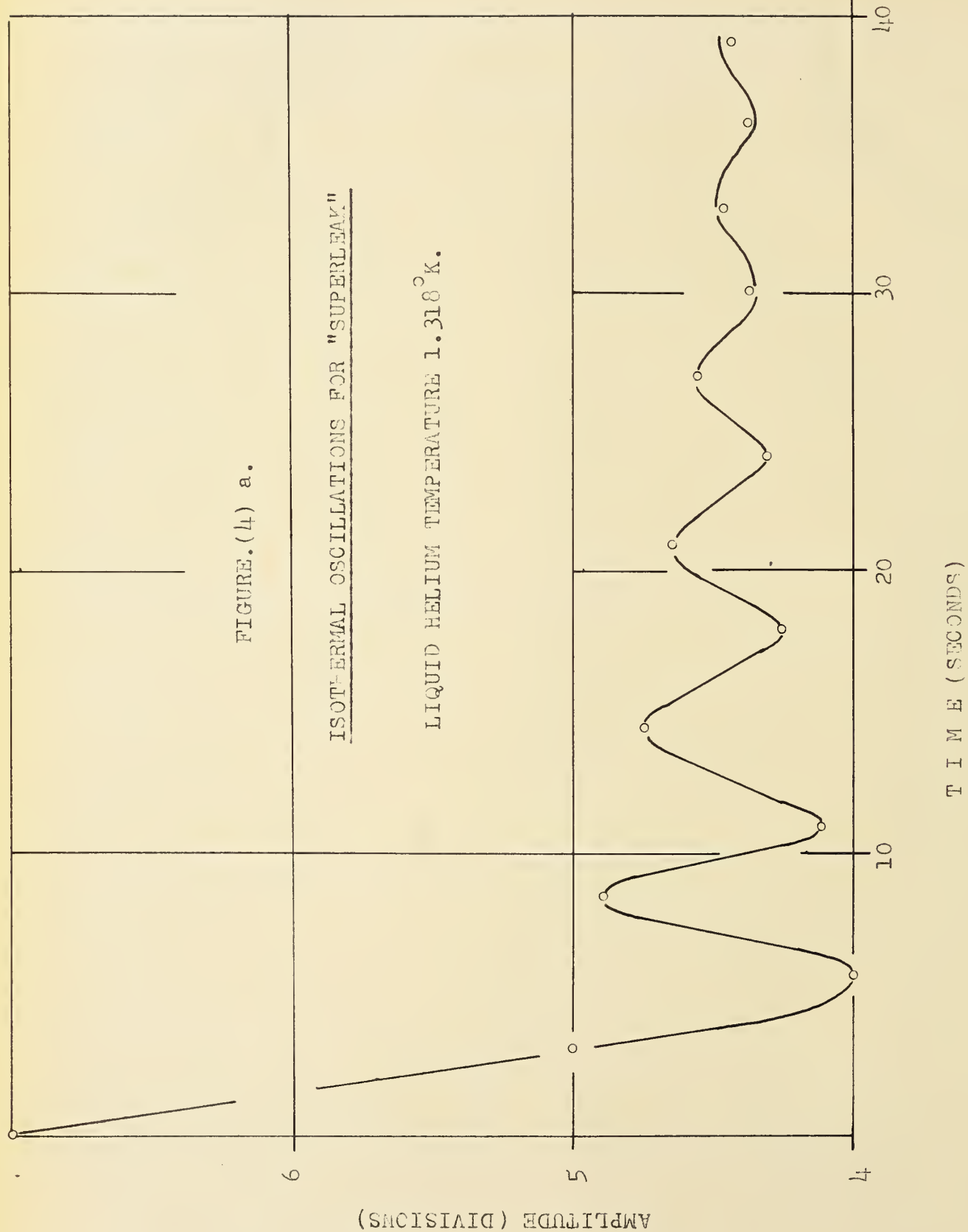
A neon lamp of 2 watts was used to illuminate the apparatus for observing the meniscus. The meniscus was observed with a cathetometer having a scale in its eye-piece (one eye-piece scale division =  $0.0583\text{ mm}$ ). The inner meniscus was allowed to settle down before the readings for this and bath level were taken.

Oscillations of the meniscus level in the capillary of the experimental





vessel (see Fig.2) were produced by raising or lowering the carriage supporting the vessel to produce a difference of several millimeters between the original meniscus level and the bath surrounding it. The liquid meniscus level sought to return to the bath level and actually overshoot the bath level position, performing oscillations about this position with <sup>a</sup>period of the order of several seconds. The position of the meniscus at regular time intervals was observed in the cathetometer. A stop watch graduated in tenths of a second was used to measure the time.



## CHAPTER IV.

### THE OBSERVATIONS ON ISOTHERMAL OSCILLATIONS

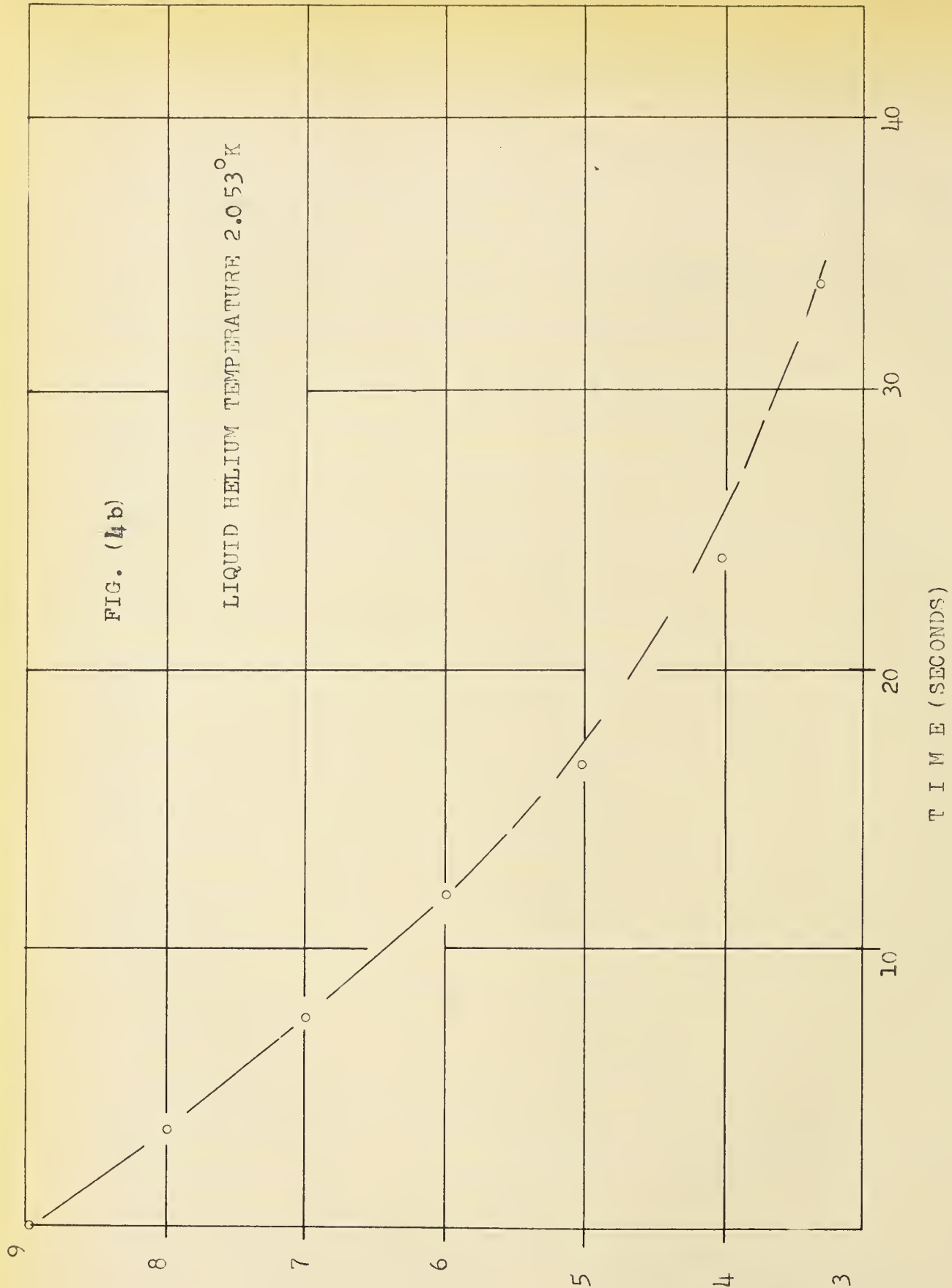
In the course of preliminary work on second sound excitation of oscillations in an adiabatic container it was decided to carry out some experiments on isothermal oscillations using a wire-filled-tube superleak. It was thought that these would provide a useful check on the superleak geometry by using the measured value of frequency of oscillations to give a value of the superleak cross-section independent of the value obtained from gas flow measurements and more likely to be the value effective in superfluid flow.

A simple copper chamber was constructed as shown in Fig.(3) and the wire-filled-tube superleak fixed in place with soft solder. Oscillations were observed with this arrangement at several temperatures in the  $1.3^{\circ}$  to  $2.1^{\circ}$  K temperature range. An example of the type of oscillations is shown in Fig.(4) a.

The interesting feature of these oscillations is that the frequency did not agree with that given by the relation (see Chapter I)

$$\omega_i = \left( \frac{\rho_s}{\rho_l} \cdot \frac{g \sigma}{l a} \right)^{1/2}$$

AMPLITUDE (DIVISIONS IN EYE-PIECE SCALE)



and that as the temperature rose, the damping of the oscillations increased to such a degree that a stage was reached where there was no observable oscillatory behaviour and the motion of the displaced meniscus in the capillary tube slowly approached the bath level with no observable overshoot (see Fig.(4) b). Table I gives the values of oscillations vs frequencies and values of the logarithmic decrement.

There was a suspicion that a leak may have appeared in the copper chamber at some time during or after the particular experimental observations were made and that the oscillations observed were not simply due to flow through the superleak. Efforts to check this have been frustrated up to the present time because the wire-filled-tube became blocked shortly after the above measurements were made. Attempts to make another superleak met with no success because of cracking of the tubing surrounding the wires after it had been drawn and annealed. Until the cause of this cracking is understood a suitable wire-filled-tube for the above measurement will not be available.

Before the difficulties with the wire-filled-tube were encountered, another experiment was performed in which oscillations were observed

T A B L E . I

TEMPERATURE (Degrees K)	FREQUENCY $\omega$ (Radians/Sec)	LOGARITHMIC DECREMENT.	THEORETICAL VALUES FOR $\omega$ (Radians/sec)
1.318°K	1.65	0.167	2.74
1.397°K	1.73	0.284	2.89
1.454°K	1.59	0.160	3.00
2.101°K	NO OSCILLATIONS		

ISOTHERMAL OSCILLATIONS FOR  
WIRE-FILLED-TUBE TYPE (SUPERLEAK)



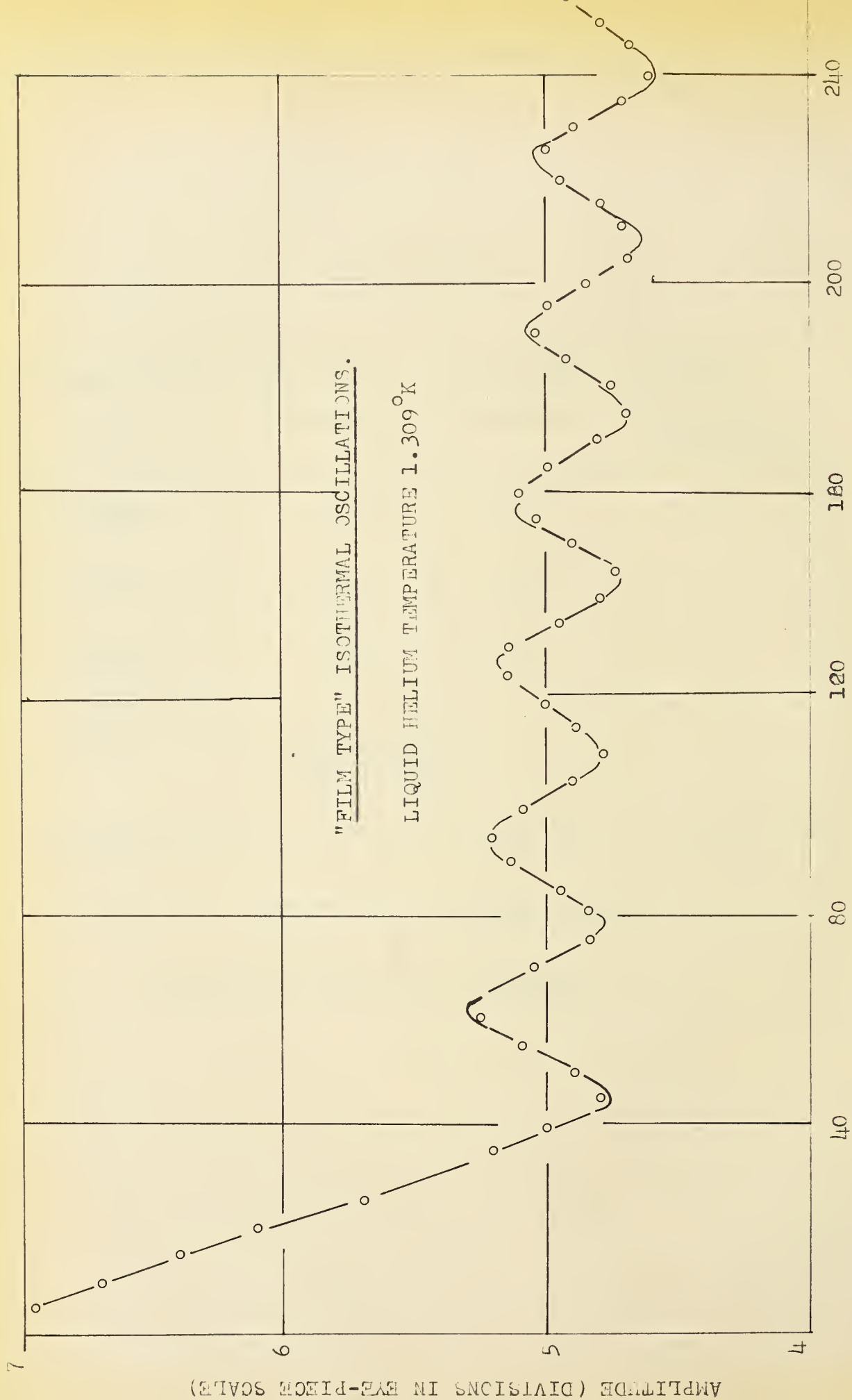


FIGURE (4) c.





T A B L E . I I

TEMPERATURE (degrees K)	FREQUENCY (Radians/sec)	LOGARITHMIC DECREMENT	THEORETICAL VALUES FOR $\omega_1^*$ (radians/sec
1.309° K	0.304	0.095	0.320
1.710° K	0.284	0.247	0.268
1.895° K	0.202	0.290	0.242

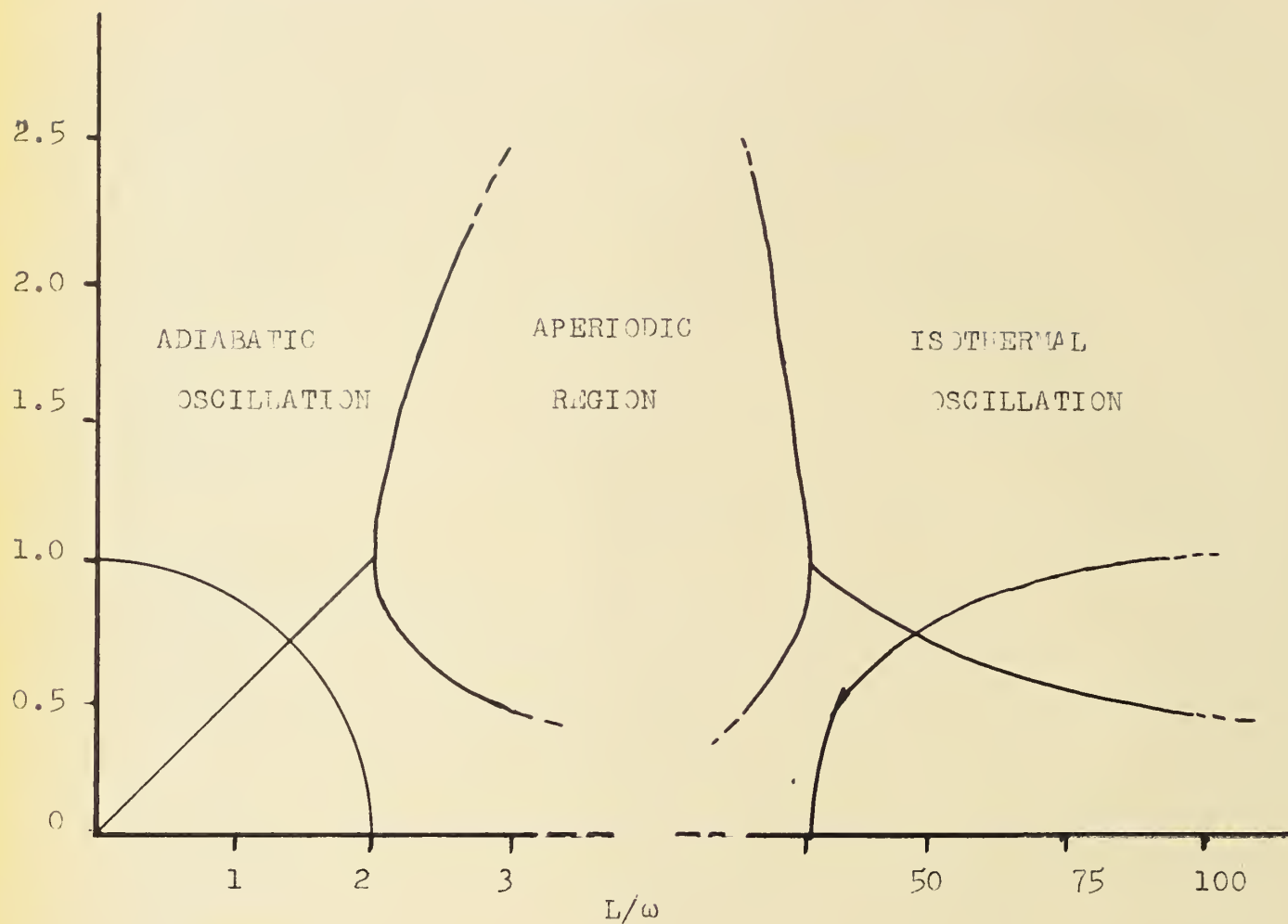
ISOTHERMAL OSCILLATIONS FOR THE FILM TYPE VESSEL.

(\* The values are estimated by making use of Atkin's(1951)  
relation given in chapter I.)

using the same copper cannister and a similar piece of Pyrex capillary (see Fig.2) but this time using the helium II film as a superleak. An example of the oscillations observed with this arrangement is shown in Fig.(4)c. It can be seen that in this case the damping of the oscillations is much less and that it does not increase so markedly with increasing temperature as in the case with the wire-filled-tube superleak. Values of frequency and logarithmic decrement were taken and are given in Table II.

The difference in damping, in the two sets of measurements, either points to a difference in the superfluid motion in the case of two superleaks or to a difference in some other physical effect which could influence the fluid motion between container and bath. Leaving aside the question of the difference in superfluid motion in the two types of superleaks, as it can not be resolved at present, we can discuss the damping produced by temperature differences between the inside and outside of the container. The effect of a non-ideal isothermal situation - i.e. lack of complete heat exchange between container and bath has been discussed fully by Robinson (1951). In his paper Robinson showed that as the thermal contact between container and bath becomes

FIGURE 4(d)



The ordinates are in units of  $\omega_a$  in and adjacent to the adiabatic region, and in units of  $\omega_i$  in and adjacent to the isothermal region.

$\omega_a$  is the adiabatic oscillation frequency

$\omega_i$  is the isothermal oscillation frequency

The above values are taken at  $T = 2^\circ \text{K}$ .

less ideal the frequency of the oscillations decreases and the damping increases. It seems very probable in the case of observations made with the wire-filled-tube that the thermal contact between inside and outside of the container was such that there was a decrease in oscillation frequency and an increase in damping and that the point was reached at about  $2^{\circ}\text{K}$  where <sup>the</sup> aperiodic region described by Robinson (1951) was reached (see Fig. 4d).

The onset of the aperiodic region at the isothermal end of the diagram is given by the expression

$$L/\omega_a = 1/2 \quad \omega_a/\omega_1 = \frac{1}{2} (1 + \alpha)^{\frac{1}{2}}$$

where  $L$  is the heat leakage factor between the container and bath. Up to about  $T = 2^{\circ}\text{K}$ ,  $\omega_a/\omega_1$  increases so that for a given value of  $L$  there is a chance that there is a temperature near  $2^{\circ}\text{K}$  where the transition to the aperiodic region sets in. It may be that the thermal contact provided through the vapour in the case of the film oscillations plays a very important part in minimising temperature differences between inside and outside, if the damping is solely due to thermal causes. The value of  $L$  can be estimated by making use of Robinson's (1951) definition

$$L = K/\rho V C$$

where  $K$  is the heat leakage between bath and container,  $\rho$  is the density of liquid helium,  $V$  is the volume of the



container and  $C$  is the specific heat of helium II. By using the dimensions given in Chapter I for  $V$  and using values for the other quantities at  $T = 2^{\circ}\text{K}$ ,  $L/\omega = 30. - 40$  . This is in the region of the value shown for the onset of aperiodic region in Fig.(4)d and does suggest that the onset of the aperiodic behaviour corresponds to the criterion given by Robinson. A much more extensive and quantitative attack would have to be made on the problem of the thermal contribution to damping, in order to rule out the possibility that there was a contribution due to the motion in the particular type of superleak employed.

It is interesting to note that in the present experiments the observed damping was very dependent on the intensity of the light source used and that only with the very weak neon light illumination were the very lightly damped oscillations in the film case observed. This variation in intensity of the light source used may account for some of the different degrees of damping reported by various experimenters (see Atkins (1960) ) even in the case of oscillations involving the helium film.

As the thermal linkage between the helium bath and experimental chamber plays an important role in deter-





mining the oscillations, there could be some excessive temperature difference due to the Kapitza boundary layer inside the vessel and outside causing an onset of the aperiodic region. According to Kapitza (1941) the thermal transfer coefficient (the number of watts that can be dissipated per  $\text{cm}^2$  per degree rise in temperature) across the solid-liquid boundary is given by

$$n = W / \Delta T$$

where "n" for small temperature difference is proportional to  $T^3$ . For a clean polished copper surface the order of magnitude of the temperature jump is given by

$$\Delta T / W \doteq 10 / T^3 \text{ watt}^{-1} \text{ cm}^2 \text{ deg.}$$

As with a decrease in temperature the Kapitza resistance increases, this onset of the aperiodic region is apparently not caused by a change in the Kapitza boundary resistance.

The reason why it was suspected that a leak developed at liquid helium temperature was that when the system was left to warm up to the room temperature the bulb B (Fig. 3) remained intact (as it could not be broken because the carriage got jammed) and did not blow up. If the inside of the experimental vessel were vacuum tight the glass bulb would have blown due to the



excessive pressure built up after the temperature rose above the  $\lambda$ -point.



PART II

GENERATION OF SECOND SOUND IN HELIUM II IN A

REVERSIBLE MANNER



CHAPTER VGENERATION OF SECOND SOUND IN HELIUM IIIN A REVERSIBLE MANNER

Second sound was predicted theoretically by Tisza (1938) and Landau (1941) and was first observed experimentally by Peshkov (1946). In the original experiment by Peshkov and in nearly all of the observations made with second sound since then, the temperature waves have been produced in the helium II by irreversible means - usually with a heater made from fine wire or a carbon film.

Second sound consists of true temperature waves in the liquid where fluctuations in temperature are a first order effect analogous to the fluctuations in pressure in ordinary or first sound (the name "second sound" is due to Landau and has remained in use). These temperature waves progress through the liquid helium in a different manner from the way a fluctuating temperature travels in a "normal" substance. For instance in second sound the heat flow (current) is proportional to the temperature excess at a point, whereas in diffusion of heat through a medium the heat flow (current) is proportional to the temperature gradient at the point in question.





Experiments have been conducted (e.g. Pellam and Marcereau, 1957) which show that a second sound wave undergoes diffraction, reflection etc., in appropriate circumstances and can be regarded as a true wave motion to which the analyses of wave optics may be applied.

It is

of interest to consider the situation when second sound is produced by means of a heater fed with a continuous alternating current. There is a fluctuating temperature signal which is superimposed on a steady background heat flow from the source. Where such continuous signals are used in an experiment, there is no longer the ideal situation of sinusoidal temperature signal travelling through a liquid essentially in thermal equilibrium but a series of positive heat pulses superimposed on a unidirectional heat flow - very definitely a non-equilibrium situation. This criticism does not apply fully to experiments where pulses of heat are fed into the helium II at sufficiently low repetition frequencies for the thermal equilibrium to be essentially maintained.

The use of a source for second sound production which avoided the production of a heat flow from the source and which produced the second sound essentially without ir-



reversible heating was first investigated by Peshkov (1948), in his method of filtration. In this method, the generator consists of a small box, one side of which is made of a copper filter with a large number of fine apertures through which only the superfluid part of liquid helium II could flow. The opposite side of this box is a steel membrane which is set into vibration as a microphone diaphragm so as to force superfluid through the filter. This produces relative motion of superfluid and normal fluid and therefore second sound. Thus in addition to the second sound this method also produces ordinary sound but the amplitude of temperature oscillations associated with the second sound is considerably greater than those of the first sound. (In his set of observations Peshkov found that at  $T = 1.63^{\circ}\text{K}$  the amplitude of temperature oscillations for ordinary sound was six times less than for the resonance of second sound.) The filtration of superfluid method becomes less effective at lower temperatures because of the preponderance of superfluid.

The only other method for the reversible production of second sound which has so far been described was introduced by Kurti and McIntosh (1955). This used a paramagnetic crystal which was subjected alternately to an increasing



and then decreasing magnetic field. This alternating magnetic field produced successive warming and coolings in the crystal and therefore gave rise to a truly sinusoidal temperature wave.

Standing waves of second sound were produced in a cavity of 3.2 cm long and 1.8 cm in diameter. At one end of the cavity was a carbon resistance thermometer serving as a detector and at the other end a freshly compressed pill of iron-ammonium-alum was placed. A small coil cooled in liquid hydrogen produced an alternating current magnetic field of amplitude up to 120 gauss at frequencies between 150 - 1200 cycles per second. The temperature excursions as measured on the carbon thermometer were of the order of  $10^{-4} \text{ }^{\circ}\text{K}$ , although the temperature amplitude of the spin system of the paramagnetic salt was of the order of  $10^{-2} \text{ }^{\circ}\text{K}$ . This is partly due to the effect of the spin lattice relaxation time but chiefly to the relatively large specific heat of liquid helium in this range. (Kurti and McIntosh 1955)

Table III (on page 40) shows the values for spin-lattice relaxation time for iron-ammonium-alum due to Benzie and Cooke (1950). With the fall of temperature the relaxation time increases, this puts an upper limit on the frequency.



TABLE III.

RELAXATION TIMES FOR IRON-AMMONIUM-ALUM (Benzie and  
Cooke, 1950)

TEMPERATURE (Degrees K)	Magnetic Field (Oersteds)	Relaxation Time (Sec x $10^{-3}$ )
2.13	285 - 968	0.2 - 0.4
0.93	285 - 968	1.5 - 0.4
1.30	400 - 795	0.3 - 0.6
0.91	343 - 841	1.4 - 3.5
2.16	400 - 748	4 - 7.5
0.94	400 - 748	44 - 80
2.16	400 - 748	1.5 - 2.5
0.96	400 - 748	10 - 20

(The values given above are for four different samples  
used by Benzie and Cooke (1950).)





This method of Kurti and McIntosh seemed to be the most appropriate way of generating the temperature waves in liquid helium II for exciting a superfluid resonator as suggested by Manchester (1955).

If a paramagnetic salt is subjected to an alternating magnetic field at liquid helium temperatures less than  $2.2^{\circ}\text{K}$  (below the  $\lambda$ -point) the magneto-caloric effect (a change in temperature which results when a body is magnetized adiabatically) is large at these temperatures and a temperature wave of sufficient amplitude can be produced. For a suitable crystal in a magnetic field of 120 gauss at  $1^{\circ}\text{K}$  this temperature amplitude can be calculated with the help of thermodynamics.

A paramagnetic salt obeys Curie's law at temperatures in the region of interest (from  $2^{\circ}\text{K}$  to just under  $1^{\circ}\text{K}$ ),

$$(1) \quad X = C/T$$

where "C" is a constant, "T" the absolute temperature and "X" the susceptibility.

Let this salt be placed in an alternating magnetic field produced by an alternating



current of frequency  $\omega$ . Thus if the magnitude of the field produced is

$$H = H_0 \sin \omega t$$

the temperature generated would have a frequency  $2\omega$  as illustrated in diagram (on page 43).

Now for a body like this from thermodynamics (see Zemansky 1957),

$$\left( \frac{\partial T}{\partial H} \right)_{S,P} = \frac{-T H (\partial X / \partial T)_{P,H}}{C_{P,H}}$$

Using (1) this reduces to

$$= - \frac{THC}{T^2 C_{P,H}}$$

$$= HC/TC_{P,H}$$

$$\therefore \partial T / \partial t = CH/TC_{P,H} (\partial H / \partial t)$$

For  $H = H_0 \sin \omega t$

$$\dot{H} = \omega H_0 \cos \omega t$$

$$\begin{aligned} d/dt(\Delta T) &= (C/TC_{P,H}) \omega H_0^2 \sin \omega t \cos \omega t \\ &= C\omega H_0^2 / 2TC_{P,H} \sin 2\omega t \end{aligned}$$

This gives

$$\Delta T = - CH_0^2 / 4TC_{P,H} \cos 2\omega t$$



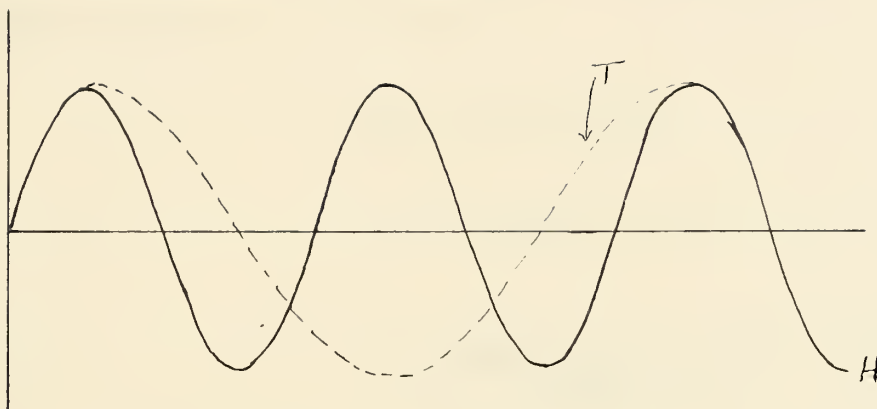


Diagram illustrating the temperature wave of frequency  $2\omega$  when the applied field was generated by applying alternating current of frequency  $\omega$ . The units used are arbitrary. (see page 42 for derivation of  $2\omega$ ).



The maximum amplitude is therefore

$$T = - CH_o^2 / 4TC_{P,H}$$

For iron-ammonium-alum, the specific heat constant  $A/R$  (deg)<sup>2</sup> (see Zemansky 1957) is given as

$$C/R = A/R \times 1/T^2$$

$$\begin{aligned} C &= (8.31/481) \times (0.0142) \times (1/T^2) \frac{\text{Joule}}{\text{gm deg}} \\ &= \frac{8.31 \times 1.42 \times 10^{-4}}{4.81} (1/T^2) \end{aligned}$$

At  $T = 1^\circ K$

$$\begin{aligned} C &\doteq 2 \times 10^{-4} \text{ joule/gm deg} \\ &= 2 \times 10^3 \text{ erg/ } K^\circ \text{ gm} \end{aligned}$$

$$\begin{aligned} \therefore \Delta T &= \frac{9 \times 10^{-3} (120)^2}{4 \times 1 \times 2 \times 10^3} \\ &= 2 \times 10^{-2} \text{ deg K.} \end{aligned}$$

(The value of  $H_o$  is taken as 120 gauss)





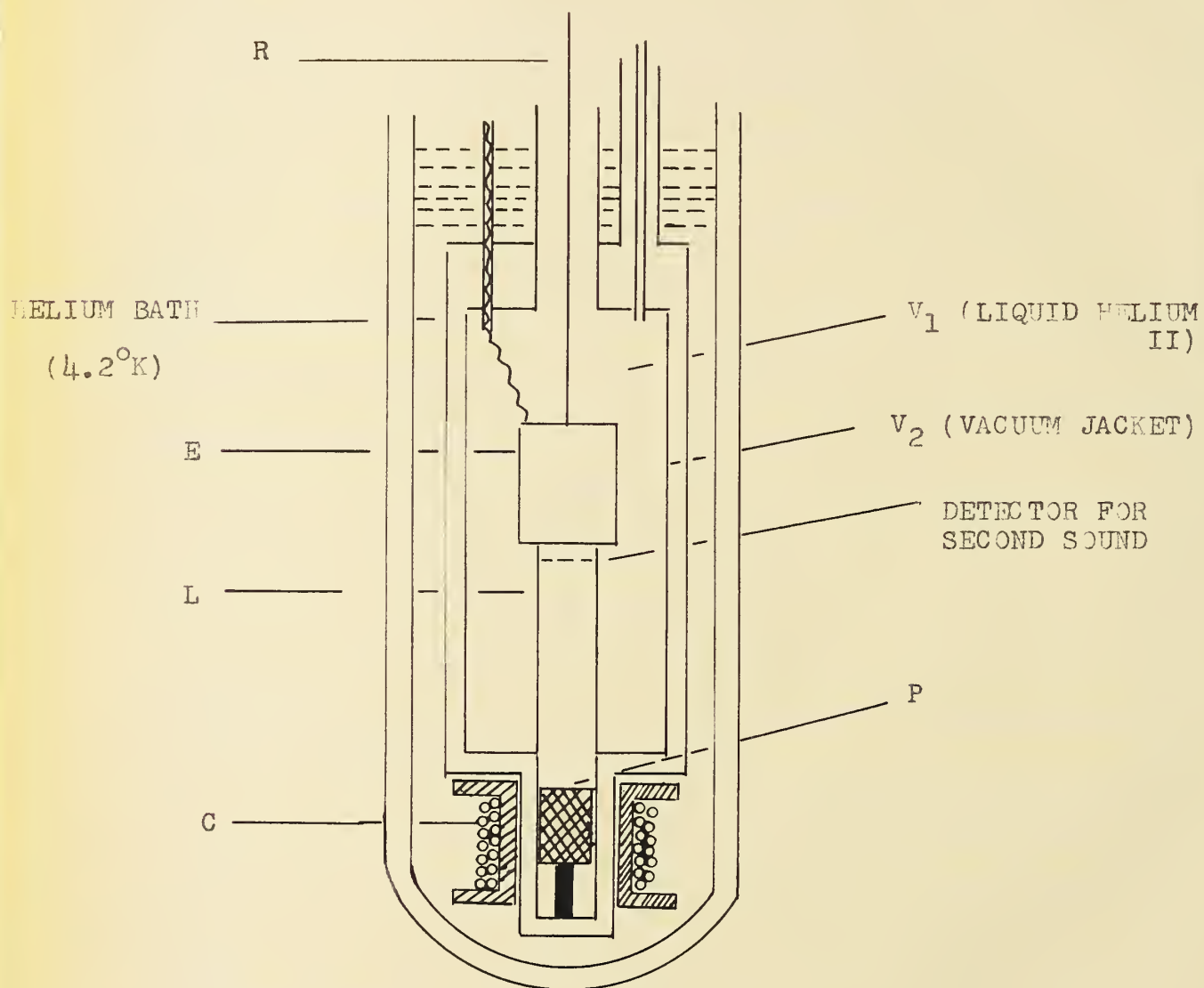
## CHAPTER VI

### THE EXPERIMENTAL APPARATUS

#### THE CRYOSTAT.

In designing the cryostat for the reversible generation of second sound due consideration had to be given to accommodating the resonator which could be observed visually and the designing of a coil to produce the required alternating current magnetic field without giving unwanted heat input. A number of attempts were made to design an appropriate coil before the present design described gave a satisfactory field for generation of temperature waves.

The main body of the cryostat was the same as discussed in part I. Few changes made in the present design are illustrated in Fig. (5). The lower part of the glass jackets —  $C_1$  and  $C_2$  (Fig. 1) was narrowed down so that the paramagnetic salt (Ferric-Ammonium-Alum) pill could be slid in easily in the inner jacket at P (Fig. 5) and a coil-former was slid on the outside jacket such that the pill was at the center of the coil. A lead wire of 0.02" diameter was wound so that when an alternating current was passed through it the required field was obtained (see Fig. 6 giving the alternating current magnetic field values). As the coil was in <sup>a</sup>liquid helium bath (4.2°K) lead wire was <sub>h</sub>

FIGURE (5)

"CRYOSTAT FOR REVERSIBLE GENERATION  
OF SECOND SOUND."

used because of its superconducting property. The heat generated by the electric current was small, presumably the only contribution was from the leads down to the coil.

A

carbon resistance thermometer was used to detect the generated thermal waves at the end of the lucite tube L. The signal was amplified (see Fig. 7) and then displayed on a cathode ray oscilloscope.

During the earlier experiments the winch W, (Fig. 1) described in part I, was replaced by a knob to which a rod R (Fig. 5) was attached. This rod helped the bridge arrangement inside the shielded box E to be properly adjusted to detect a signal. All pumping tubes were the same as those described in part I. Pressure measurements were also done in the manner discussed before.

#### PARAMAGNETIC SALT PILL.

A pill of the paramagnetic salt (Ferric-Ammonium-Alum) 1.7 cm in diameter and 2.3 cm long was made by compressing the finely powdered salt at a pressure of 1200 lb/inch<sup>2</sup>. It is well known that the exposed surfaces of such a salt pill disintegrate rapidly due to the loss of water of crystallization. So



to have the least area of the surface exposed a coating of nail varnish was used and for added protection the salt pill was put in a thin lucite casing before it was slid in the narrow part P (Fig.5) of the inner copper-glass seal.

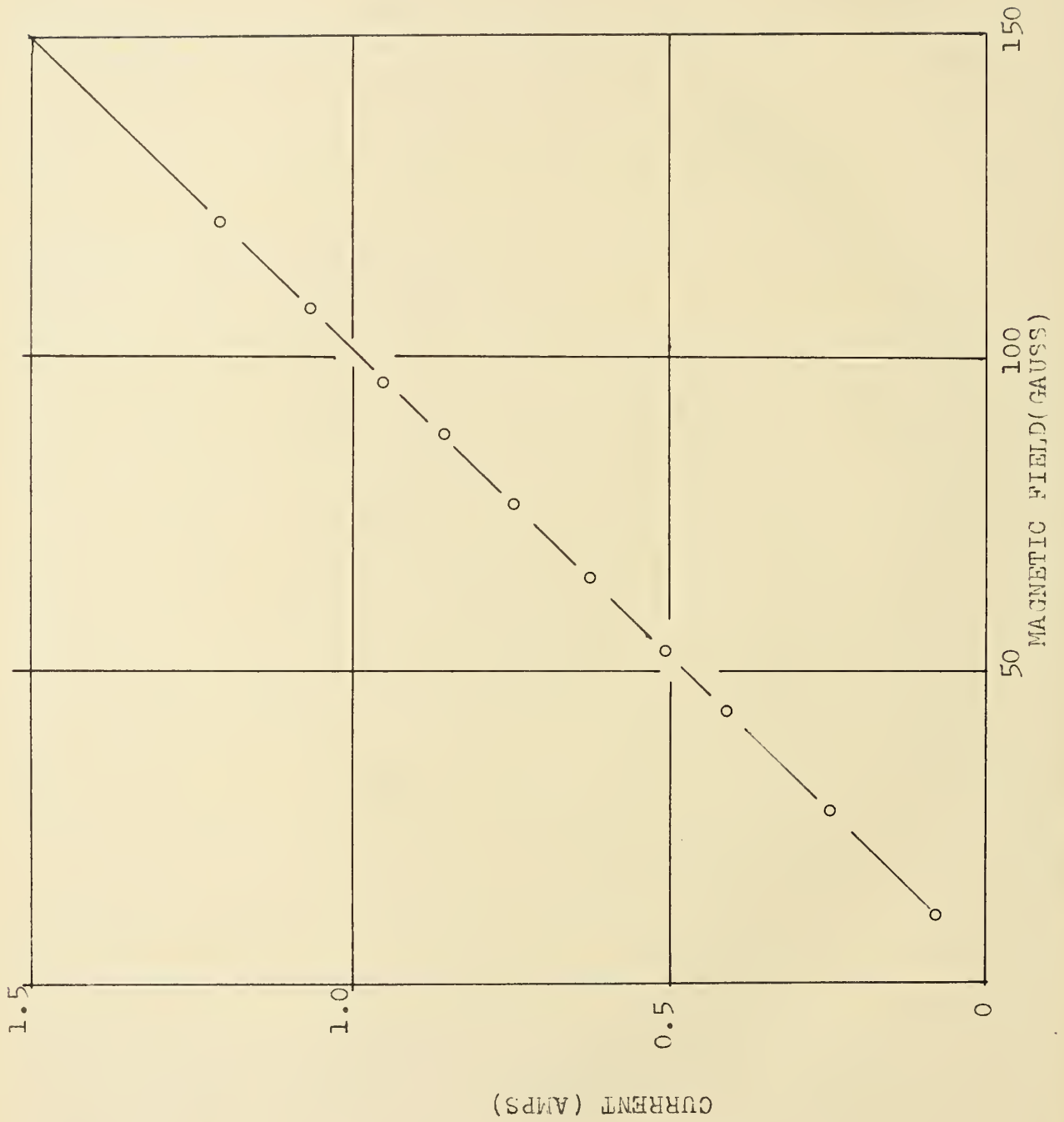
#### COIL FOR PRODUCING A/C MAGNETIC FIELD.

In the earlier attempts of making a coil to produce an alternating current magnetic field up to 120 gauss it was thought that a Helmholtz set-up of 700 turns of B and S gauge # 28 copper magnet wire should give the required field when an audio-amplifier was used as current supply. To avoid excessive heating in the helium bath ( $4.2^{\circ}\text{K}$ ) the best way to wind the coil was to keep it in the liquid air dewar. But this proved of no use as the field produced was nowhere close to the required one for generating the thermal waves. After a couple of more unsuccessful attempts it was decided to use lead wire and keep it in the helium bath.

Pure lead (99.997 %) was used to extrude the 0.02" diameter wire. A special extruder was made for this purpose and the facilities of the Metallurgy Department at the University of Alberta were used to make

FIGURE.(6)

CURRENT vs FIELD FOR THE LEAD WIRE COIL



the wire. The G.E. (baking type) varnish was used for insulation by electrically heating the wire. After winding the wire for a few turns on the hard wood coilformer, it was noticed that the varnish blobs made the turns uneven so the nail varnish was used for insulation. This was tried at helium temperatures and no obvious heating effect was noticed in the experimental chamber though the coil was in the helium bath at  $4.2^{\circ}\text{K}$ .

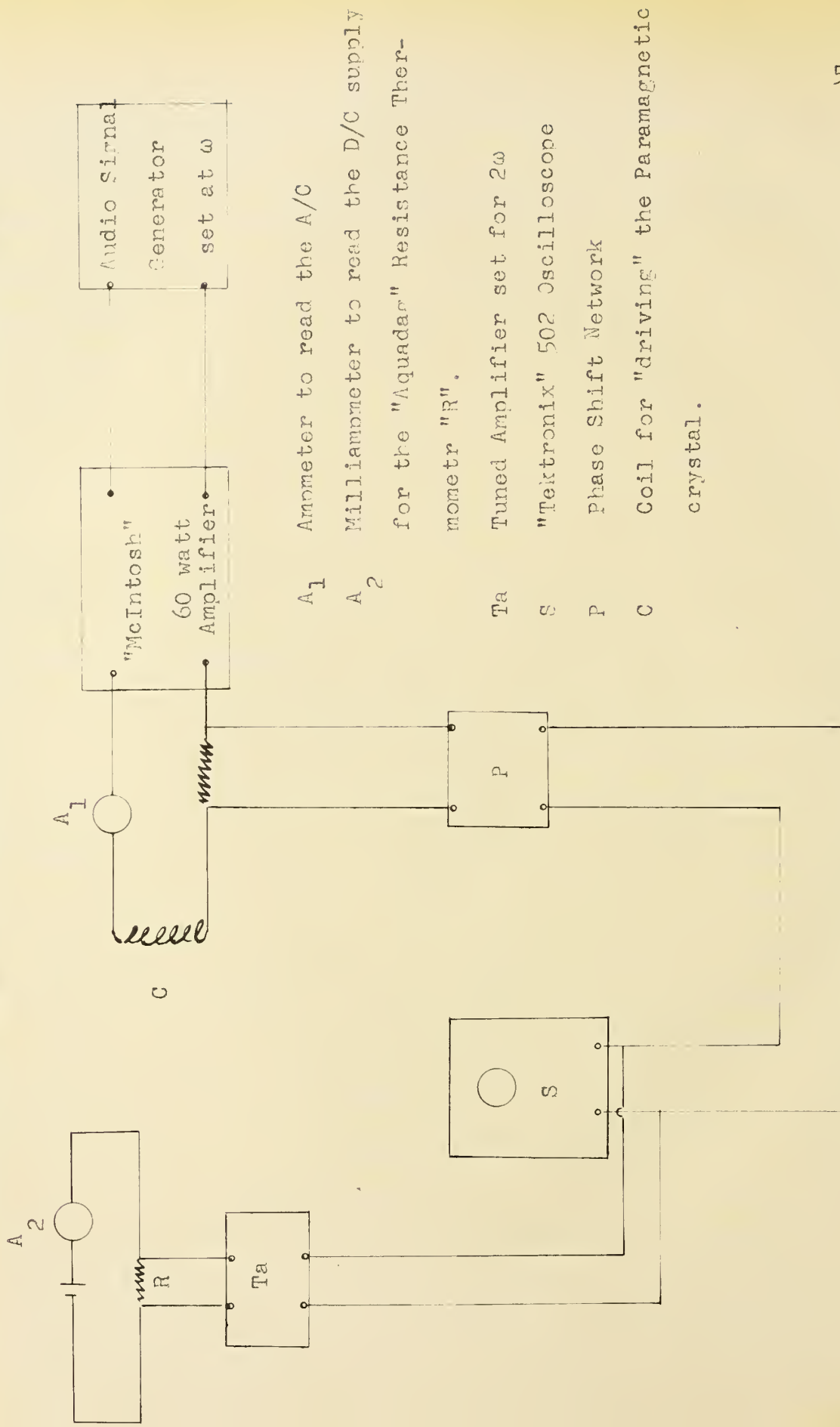
#### ELECTRICAL CIRCUITS.

The coil for supplying the alternating magnetic field which produced the temperature oscillations in the paramagnetic crystal was connected to the output of a 60 watt amplifier which was driven by an audio signal generator (see Fig. 6 for the magnetic field produced). The circulating current in the coil could be made as high as 2 amperes which corresponds to a field of 200 gauss at the position of the crystal (i.e. 100 gauss/amp.). This should be sufficient to provide a temperature signal of amplitude great enough to be detected (see Fig. 7 on page 50). The wires leading to the coil and part of the coilformer were covered with lead foil to provide magnetic shielding so that a minimum of stray magnetic field



FIGURE (7)

BLOCK DIAGRAM FOR THE GENERATION OF SECOND SOUND REVERSIBLY. (ELECTRICAL CIRCUITS)





would affect the thermometer circuit.

One face of the paramagnetic crystal of powdered and compressed ferric-ammonium-alum formed one end of a small cavity in which the temperature oscillations were excited. The other end of the cavity was formed by a circular disc of stiff paper coated with aquadag which formed the detecting resistance thermometer (see Appendix II for description of the resistance thermometer). The resistance thermometer was fed by a small D.C. current of up to a few milliamps and <sup>the</sup> fluctuating potential difference across the thermometer due to the incoming second sound signal was fed into the primary of a step up transformer with a ratio of 25/50. The output of this transformer was fed into a two stage audio pre-amplifier and from there to the input of an oscilloscope. The pre-amplifier behaved as a tuned amplifier which could be set at frequencies between 100 and approximately 1500 cycles per second. This was accomplished by having a Wien bridge in a feedback loop between the two stages. The tuned amplifier was used in order to eliminate as far as possible the effects due to stray signals picked up in the thermometer circuit - these arose mostly from the alternating magnetic field used to excite the paramagnetic crystal and from the 60 cycle mains supply.



As the tuned amplifier was set at twice the frequency of the signal fed to the magnet coil in order to detect the second sound signal (see page 50) and lead shielding was used as much as possible, there was not a great effect from the magnetic field of the driving coil. At times a phase shift network was used to feed a small signal from the magnet circuit into the amplifier circuit with such a phase that the stray signal seen on the oscilloscope screen was cancelled out.

A more important source of interference was the 60 cycle mains supply even though a tuned amplifier was used. The "noise" produced by this source was harder to keep down to a low enough level. It was not a pure signal as in the case of the magnetic field and not as well localised. With the present arrangement second sound has not been detected and the principal problem is to reduce the electrical background sufficiently to observe the signal clearly at the expected level.



## APPENDIX I.

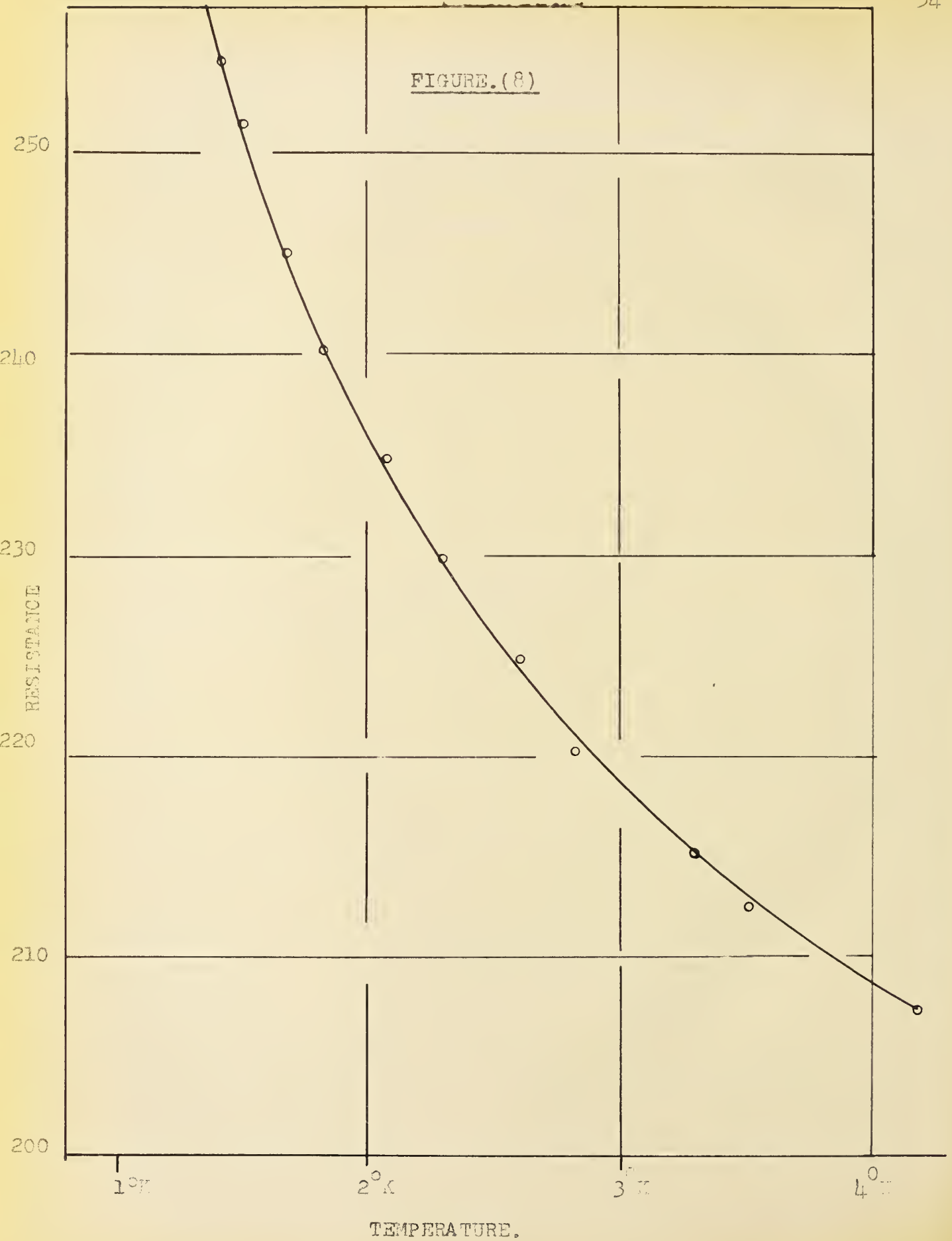
### THE WIRE-FILLED-TUBE (SUPERLEAK)

Allen and Misener (1930)

first introduced these tubes and since then they have been extensively used by Brown and Mendelssohn (1947) and Manchester and Brown (1957). During the first few runs the wire-filled-tube used was the one used by Manchester (1955) in his experiments on adiabatic oscillations. In the later runs it was decided to make a new tube as this old one was suspected to be blocked. The techniques used were those suggested by Manchester.

A bundle of fine constantan wire (1200 wires of 0.002" diameter) was placed in a cupro-nickel tube (0.130" O.D. and 11 $\frac{1}{4}$ " long) and this tube was drawn through a succession of holes in a steel-die plate. As the tube got thinner after each drawing, the space between the wires got smaller and smaller. After the tube had been drawn through six holes (die # 34, 0.103" O.D.) helium gas was passed through it to check that it was not blocked and estimate the cross-sectional area of the channels.

In order to avoid the splitting or cracking of the tube it was annealed after it had been drawn 2-3 times. In spite of all this care the tube cracked after being drawn through die # 34 (O.D 0.103").



For measuring the channel cross-section, the tube after being sealed to the experimental vessel was placed in a glass cone which was sealed into an arrangement where helium gas could be supplied to one end of tube and the other was connected to a vacuum system equipped with a McLeod gauge. The rate of rise of pressure was measured in the vacuum system when the pump was shut off and the helium gas was flowing in the tube. This helped in finding the volume flow rate, and estimating the channel width by applying the Poiseuille's relation for streamline in a channel of rectangular cross-sectional area with parallel sides viz.,

$$h^3 = 12\eta l \dot{V} / \Delta p a$$

where " $\eta$ " is the viscosity of gas, " $l$ " the length of channel,  $\dot{V}$  the volume rate flow and " $a$ " channel breadth.



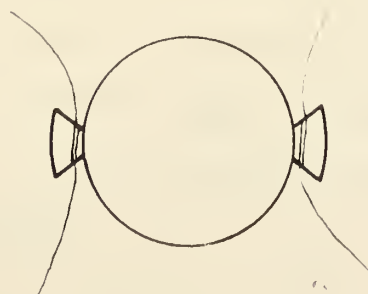


## APPENDIX II

Carbon (Aquadag) home made resistance thermometers were used for detecting the second sound generated as similar resistors have been used by many experimenters (Laredo 1955, Kramers 1955) satisfactorily. Aquadag (colloidal carbon) was painted on circular thin cardboard sheet (2.5 cm in diameter). The leads were of fine constantan wire (B and S gauge # 38) tightly wound around two small symmetrical wings of sheet (see Fig.9). To obtain good thermal contact a layer of aquadag was painted on the wires in contact with the cardboard sheet, as well. Nail varnish was used to keep the constantan wire in position.

A calibration curve for the aquadag resistor is shown in Fig.(8). The sensitivity (defined as  $\frac{1}{R} \frac{dR}{dT}$ , where  $R$  is the resistance in ohms and the temperature  $T$  in absolute degrees) for the resistor was  $0.076 \frac{\text{deg}^{-1}}{\text{K}}$  at  $2.5^{\circ}\text{K}$ .

FIGURE.(9)



Aquadag painted carbon resistance.



# B I B L I O G R A P H Y

- Allen J.F and Jones H. Nature (London), 141, 243, (1938)
- Allen J.F and Misener A.D Pro.Camb.Phil.Soc. 34, 299, (1938)
- Atkins K.R Pro.Roy.Soc.(London) A 203, 119, (1950)
- Atkins K.R Int.Con.Low Temp.Phys.(1960)
- Benzie R.J and Cooke A.W Pro.Phys.Soc.(London), A63, 201, (1950)
- Brown J.B and Mendelssohn K. Nature(London), 160, 670, (1947)
- Clement J.R , Logan J.K and Gaffney J. Phys.Rev.100, 743, (1955)
- Daunt J.G and Smith R.S Rev.Mod.Phys., 26, 172, (1954)
- Kapitza P.L Nature (London) 141, 74, (1941)
- Kapitza P.L J.Phys.(Moscow) 4, 181, (1941)
- Kramers H.C et al. Physica, 20, 743, (1954)
- Kramers H.C et al Pro.Int.Con.Low Temp.Phys.(Toronto) 562  
(1960).
- Kurti N. and McIntosh J. Phil.Mag.(London), 46, 104, (1955)
- Landau L.D J.Phys.(Moscow), 5, 71, (1941)
- Landau L.D J.Phys.(Moscow), 11, 91, (1947)
- Laredo S.J Pro.Roy.Soc.(London), A229, 473, (1955)
- Manchester F.D Can.J.Phys.( 33, 1148, (1955)
- Manchester F.D and Brown J.B Can.J.Phys. 35, 483, (1957)
- Pellam J.R and Marcereau R. Phys.Rev.106, 1113, (1957)
- Peshkov V.P J.Phys.(Moscow) 131, 8, (1946)
- Peshkov V.P Report Int.Con.Physical Society (London)  
2, 19, (1946)



Peshkov V.P J.Phys.(Moscow),18,857, (1948)

Robinson J.E Phys.Rev. 82,440, (1951)

Tisza L. Nature (London) 141,913, (1938)

Tisza L. Phys.Rev. 72,838, (1947)

Zemansky M.W "Heat and Thermodynamics." (1957)











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